Problem 3.9

(a) Cite a Hamiltonian from Chapter 2 (*other* than the harmonic oscillator) that has only a *discrete* spectrum.

(b) Cite a Hamiltonian from Chapter 2 (*other* than the free particle) that has only a *continuous* spectrum.

(c) Cite a Hamiltonian from Chapter 2 (*other* than the finite square well) that has both a discrete and a continuous part to its spectrum.

Solution

Part (a)

The Hamiltonian for the infinite square well in Section 2.2 has only a discrete spectrum (with normalizable solutions):

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x),$$

where

$$V(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq a \\ \infty & \text{otherwise} \end{cases}.$$

Part (b)

The Hamiltonian for the delta-function barrier in Section 2.5 has only a continuous spectrum (with non-normalizable solutions):

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x),$$

where

$$V(x) = \alpha\delta(x), \quad \alpha > 0.$$

Part (c)

The Hamiltonian for the delta-function well in Section 2.5 has both a discrete part (with normalizable solutions) and a continuous part (with non-normalizable solutions) to its spectrum:

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x),$$

where

$$V(x) = -\alpha\delta(x), \quad \alpha > 0.$$